A BEHAVIOURAL FINANCE APPROACH WITH FUNDAMENTALISTS AND CHARTISTS IN THE GOLD MARKET

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ABSTRACT. The substantial increase in the price of gold in recent years appear to be a candidate for a potential asset price 'bubble'. This observed bubble-like behaviour suggests that chartists (feedback traders) are highly active in the gold market and hence, motivated by these observations, this paper develops and tests empirically several models incorporating heterogeneous agents, specifically fundamentalists and chartists, for the gold market. The empirical results show that the past trend in the price of gold is consistent with the presence of chartists who follow trends as opposed to fundamentalists who use other criteria. Technically this paper is a further step toward providing an empirical foundation for certain assumptions used in the heterogeneous agents literature. For example, the empirical results presented in this paper identify economically and statistically significant switching specifications and variables, that are generally only introduced ad-hoc.

JEL classification: C51; D03; G12

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1. INTRODUCTION

Gold prices have exhibited an extremely strong positive trend over the last five years with gold recently trading well above 1,600 US dollar per troy ounce. Comparing this to the relatively stable period for the thirty years previous there has been much discussion about the cause of this upward trend. This paper attempts to shed some light onto this question through the use of a heterogeneous agents model for the gold market. To our knowledge, this is the first such empirical work focusing on the gold market.

The use of a heterogeneous agents model can be justified by the observation that the historical gold price has exhibited a strong and relatively long positive trend which can be explained with

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chartists or feedback traders buying an asset whose price increased in the past thereby enforcing or prolonging the past trend. This behaviour is in contrast to a fundamentalist, i.e. a trader who buys an asset when it is below the fundamental price and sell the asset when it is above the fundamental price. A long and positive price trend can thus only be explained by a continuously changing fundamental price or the presence of chartists. We think that the latter, i.e. the presence of chartists is a more plausible assumption than a continuously changing fundamental price.[1]

Specifically, we develop several parsimonious heterogeneous agents models for the gold market and map the time-dependent switching mechanisms used in these models onto three well known and robust numerical estimation procedures (threshold autoregressive (TAR), smooth-transition autoregressive (STAR) and Markov regime switching models) with the aim of providing empirical information on the price forming mechanisms at work in the gold market in recent years.

Despite their theoretically appealing features, there are many different alternative model specifications available in the heterogeneous agents literature each producing different results. With this in mind, we attempt to maintain as much generality as possible, within our chosen class of model, to allow for the testing of many different model specifications, ultimately enabling us to let the data tell us the correct specification. In addition, note that we do not consider exogenous variables in this paper but only use endogenous, i.e. lagged log-prices, simple or squared log-price differences. While the use of exogenous variables offer a large set of modelling opportunities the choice of such variables is not straightforward and we therefore restrict ourselves to a 'classical' time-series analysis.

This paper aims to provide a better understanding of the deviations of the price of gold from its long-run mean and to shed some theoretical light onto periods of persistent price changes. In doing so, we also aim to better understand the role of the 'fundamental value' of gold and similar assets that do not provide any future cash flow and so do not have a consensus theoretical fundamental value as in the equity or housing market.

[1]It is often argued that the increased demand from emerging countries especially India and China caused the recent trend in the price of gold. While we agree that emerging market demand for gold played an important role in the past we do not believe that such a change in demand for an asset should result in the price process observed recently in the gold market, i.e. a positive price trend over a relatively long time period. If investors anticipate the increased demand from emerging countries, the price should change almost immediately but not over a period spanning several years.

The empirical findings are in line with predictions from heterogeneous agent theory, in particular we observe periods of mean reversion and periods of explosive price behaviour in the gold market, hence providing evidence of the existence of heterogeneous agents (fundamentalists and chartists) in the gold market. Furthermore, identifying those periods when speculators or chartists have become active in the gold market allows us to ascribe an intuitive economic explanation for the dynamics of historical gold prices that are often difficult to explain with standard (homogeneous) financial theory.

Many commentators have identified the recent trend in gold prices as a bubble[2] however many others have asked the question as to whether or not there has been a structural change in the prices and volumes in commodity markets in recent years.[3] There has been a dramatic increase in the presence of institutional investors or financial institutions in commodity markets who use this market for the purpose of asset management. Therefore the period from 2004 until recently is sometimes referred to as a period of financialization of commodities (e.g. see Tang and Xiong, 2010). The publication of empirical evidence of a low correlation of commodities with other, more traditional, assets such as stocks and bonds has triggered an increased presence of investors in these markets. In addition, the introduction of exchange-traded funds around 2004 has further augmented this trend by providing both institutional and retail investors additional opportunities to obtain exposure to commodities such as gold. Does this mean that the commodity market and the gold market in particular has lost its character as an alternative asset? The chartist-fundamentalist approach could offer a new and promising alterative behavioural perspective on historical gold price dynamics.

1.1. **Theoretical literature.** Models of asset price dynamics based on the interaction of heterogeneous agents have become increasingly popular in recent years.[4] These models are appealing since they allow for more flexibility in the modelling of investors behaviour than classical expected utility theory would allow. Furthermore, even some of the simpler models in this class appear to be able to explain well many of the stylized empirical facts of observed asset price

[2]See, for example, "Gold price bubble a 'high probability' says Deutsche Bank", *Financial Times*, January 13, 2011 (`http://www.ft.com/cms/s/0/2e36ccf4-1f33-11e0-8c1c-00144feab49a.html`).
[3]See, for example, "Gilt-edged argument", *The Economist*, April 28, 2011 (`http://www.economist.com/node/18620836`).
[4]For a detailed survey of the history and state-of-the-art of heterogeneous agents models see Hommes (2006) or more recently Chiarella et al. (2009).

dynamics. Essentially these models posit that asset prices are driven to some degree by an endogenous nonlinear law of motion.

In the literature to date, this nonlinearity has been introduced into the price dynamics in two alternative ways. One imposes nonlinear demand functions and hence nonlinear trading rules on the heterogenous agents acting in the market, for example see Day and Huang (1990), Chiarella (1992) and Farmer and Joshi (2002). Furthermore, in an attempt to ground this nonlinear demand in classical economic theory, this demand is often derived from classical utility maximization (see for example Chiarella et al., 2007). Alternatively, one can impose simple linear demand functions on the heterogeneous agents but allow the relative weighting of each heterogenous agent's demand to vary through time according to a given rule. The most common interpretation of these time-varying weights being that the agents are allowed to 'switch' between strategies. This idea, influenced by the discrete choice modelling literature (see Manski and McFadden, 1981; Anderson et al., 1993), was developed by Brock and Hommes (1997, 1998) who coined it the 'adaptive belief system'. On a related topic, and as supporting evidence for the existence of switching in financial markets, Abreu and Brunnermeier (2003) show that difference of opinion about timing of bubbles amongst fundamentalists can allow noise traders to create a bubble. More importantly, they also show that it could be optimal for fundamentalists to switch to a positive-feedback strategy in this setting.

Obviously one can combine both of the above approaches for a richer, more complex, heterogeneous agents model. In this paper, we adopt the second (linear demand and switching) approach. Furthermore, we consider two agent types, fundamentalists (or value investors) and chartists (or trend followers), to model the price dynamics of gold through time. Chartists may also be thought of as momentum traders and fundamentalists as contrarian traders. Whatever their name, these two types of agents are the most widely used in the literature on heterogeneous agent based models. In the majority of the existing theoretical heterogeneous agents literature, it is found that chartists tend to destabilize markets (increase volatility), whereas fundamentalists act as a stabilizing force on price dynamics.

If there are more fundamentalists than chartists in a market at any given time then prices are pushed toward fundamentals. Such directional price movements, in turn, make the market more attractive to chartists, luring more and more of them to enter the market, reinforcing the observed trend. Thus the actions (and interaction) of the two types of agents through time can lead to periods of both mean reverting and explosive behaviour.[5]

It is worth noting at this stage that the interpretation of chartists and fundamentalists need not be distinct traders but different motives of a single individual for changing his holdings. Either way the mechanism is clear.

1.2. **Empirical literature.** When it comes to the empirical estimation of the heterogeneous agents models described above, there appears to have been limited work done in the literature to date and empirical guidance on the choice of the parameters employed in these models has been limited. This sentiment is echoed in Chiarella et al. (2009) who state in their conclusions that more work needs to be done on estimating the models surveyed in their paper. The relatively small number of empirical studies is no doubt due to the highly nonlinear nature of the models and the resulting econometric issues. In particular the nonlinear switching mechanism poses some interesting econometric difficulties in their estimation – these will be elaborated on in Section 3.

The following attempts to survey the existing literature on the estimation of heterogeneous agents models (henceforth HAMs), focusing in particular on the estimation of the nonlinear switching models, the class of models in which we find ourselves. The most striking observation from the empirical literature comes from Amilon (2008), who states that the majority of the stylized facts produced by HAMs are purely an artifact of adding noise to the price process in an 'additive' rather than a 'multiplicative' way. When done correctly (multiplicative), he states, the effects all but disappear. Amilon (2008) also suggests that fast switching between agent types appears to be necessary to fit these models.[6] Manzan and Westerhoff (2007) find statistical evidence for regime-switching behaviour of chartists in foreign exchange markets. To be more precise, chartists turn from positive- to negative-feedback trading when the most recent absolute price change has exceeded a certain threshold.

In order to avoid the need for estimating the nonlinear switching mechanism directly there have been numerous approaches employed in the existing literature. One intuitive approach used has been to estimate the model using a Markov regime-switching approach (see for example Vigfusson, 1997; Ahrens and Reitz, 2005). We shall also employ this method here, in the

[5]For an extensive collection of possible rules used by chartists see Murphy (1999).
[6]Note that we will also see this in our results later.

context of the gold market, with some interesting results. Other 'indirect' methods include the use of filtering techniques (see Baak, 1999; Chavas, 2000) and simulation based estimation such as those employed in Gilli and Winker (2001, 2004) who estimate a HAM by minimizing a particular loss function (dependent on the statistical properties of the simulated data) by adjusting the coefficients of the model. Care needs to be taken in such approaches since such objective functions are generally not globally convex. Furthermore, Alfarano et al. (2005, 2006) take advantage of a derived closed-form expression for the stationary distribution of returns for a particular class of HAM to estimate the distribution of agents within a given market.

There has been a few attempts to 'directly' estimate the nonlinear switching mechanisms. Boswijk et al. (2007) estimate a version of the Brock and Hommes (1997) model directly for S&P500 data using nonlinear least squares and Reitz and Westerhoff (2003, 2005, 2007) estimate a model of chartists and fundamentalists with a switching mechanism for daily exchange rate data. More recently, de Jong et al. (2010), have also estimated heterogeneous agents models for the exchange-rate dynamics of the European Monetary System (EMS), using quasi-maximum likelihood techniques. In addition, similar techniques are employed in Kouwenberg and Zwinkels (2010) to investigate the behaviour of the US housing market market over the last 50 years and in ter Ellen and Zwinkels (2010) when considering oil price dynamics.

The majority of the above papers find some evidence of trader heterogeneity and switching. However, there is no study which analyzes a heterogeneous model for the gold market despite the strong positive trend and the bubble-like behaviour in the gold market in recent years. Since it appears unlikely that the last 30 years of historical gold prices, including the substantial change in the price process in recent years, can be explained with one representative agent, there appears strong motivation for such an analysis.

1.3. **Is there a fundamental price of gold?** As stated above, the recent trend in the price of gold (see Figure 1) bears some resemblance to a typical bubble. While this possibility would justify the use of a heterogeneous agents model with fundamental traders and chartists (speculators), it is also possible that the fundamental price of gold has changed in recent years. The following discusses the role of the fundamental price.

Gold exhibits various roles in the global economy. It is used as a financial asset to diversify risk and works as a safe haven during financial turmoil. In equity markets fundamentalists base their estimate of the fundamental value on a detailed analysis of dividends and earnings forecasts or more simply on the expected dividend stream. Since gold does not provide any future cash flow, other than possible capital gain, what can we say about the fundamental price of gold?

One argument is that since it pays no dividends its real value is simply a constant. This belief leads many investors to invest in gold as a hedge against future inflation; such investors often do not want to hold paper money in case of future inflation so they choose to invest in gold. The reasoning is as follows. Since gold's value is denominated in government currencies, if the real value of gold remains constant, its value will tend to increase if the value of the currency of denomination is eroded by inflation. Thus, gold (and other commodities) can provide a natural hedge against inflation.

An alternative argument is that, since gold is used as a store of value and a safe haven for investors, the fundamental price of gold could depend on the economic state of other markets. Empirical evidence suggests that agents may perceive fundamental values according to the 'anchor and adjustment' heuristic. Tversky and Kahneman (1974) show experimentally that people make estimates of value by setting an instinctive initial value and then adjusting this value using other considerations to achieve the final answer. However, adjustments are typically insufficient, implying biased estimates towards these initial values.

We think that it is safe to argue that the extreme change in the price of gold from values around 400 US dollars to values above $1,600$ US dollars between 2005 and 2011 cannot be solely explained with a change in the fundamental price of gold. If the price of gold includes historical inflation and future expected inflation, future expected inflation rates must be very high to fundamentally explain a price of gold above $1,600$ US dollars. In contrast, we argue that the recent evolution of the gold price cannot be explained with one homogeneous agent but only with two, heterogeneous, agents. Only if chartists are added to a model with fundamentalists, the recent trend in the price of gold can be explained.

In most equity markets, one of the standard stylized facts observed is that they are prone to periods of bubbles and crashes. Since a bubble is often defined as a prolonged period where prices are above their fundamental price, it is hard to assess the existence of bubbles in a market where there is no fundamental price or a theoretical model to determine such a price. Due to these problems we avoid estimating or calibrating the price of gold *a priori* choosing instead to estimate the price conditional on certain regimes determined by the presence of chartists and fundamentalists. In addition, we rather follow the approach introduced by Phillips and Yu (2011) in which a bubble is defined as an explosive price process.

This paper is organized as follows. Section 2 introduces the heterogeneous agents models employed in this paper and Section 3 discusses their econometric estimation framework and discusses in detail the econometric issues associated with the estimation of these highly nonlinear models. Section 4 presents the estimation results and interprets these results in the context of the economic heterogeneous agents models. Finally Section 5 concludes.

2. Heterogeneous Agents Models

Following Farmer and Joshi (2002) and He and Westerhoff (2005)[7] we assume a log-linear price impact function which results in the following structural model for the log-price P

$$P_{t+1} = P_t + a\left(D_t^M + D_t^F + D_t^C\right) \tag{1}$$

where D denotes excess demand, the superscripts M, F and C denote market (real economy) demand, fundamentalist demand and chartist demand, respectively.[8] The parameter a is a positive price adjustment coefficient.[9] We assume that the excess demand from the real economy (i.e. producers and suppliers) is random and thus we set $D_t^M = e_t$ where e is normally distributed with mean zero and variance σ^2. For the remaining demands we use a reduced form model[10]

$$\begin{aligned} D_t^F &= a_F W_t^F \left(E_t^F[P_{t+1}] - P_t\right), \\ D_t^C &= a_C W_t^C \left(E_t^C[P_{t+1}] - P_t\right), \end{aligned}$$

where a_F and a_C are positive reaction coefficients, $E_t^i[P_{t+1}]$ for $i = F, C$ are the expectations of the next periods price of the fundamentalist and chartist respectively, and crucially, W^F and

[7]He and Westerhoff (2005) propose a similar behavioural heterogeneous agents model to the one proposed here, also in the context of the commodities market, however their focus is on exploring the effectiveness of price stabilization schemes such as price limiters. Interestingly they find that simple measures to control prices have surprising consequences in a nonlinear world.

[8]Note that implicit in the model is the existence of a market maker who absorbs temporary imbalances in excess demand from the other agent types, lowering the price when they have to buy and raising it when they have to sell, (see Farmer and Joshi, 2002)

[9]We will see later that we may set $a \equiv 1$ without loss of generality.

[10]The model we employ here can be considered a behavioural finance model. The rules used by the fundamentalists and chartists to determine demand are simply rules of thumb or heuristics rather than being derived by classical expected utility maximization. However, Hommes (2001) showed that these simple demand functions are equivalent to those of a 'myopic' mean-variance maximizer. For examples of agents demand functions derived from utility maximization see Chiarella et al. (2007, 2009).

W^C are (possibly) time varying weights to incorporate switching between strategies; included to capture the time-dependent nature of the relative importance of each agent type in the market at any given time. Furthermore we assume that

$$E_t^F[P_{t+1}] = P_t + b_F(F - P_t) \tag{2}$$

hence fundamentalists expect prices to revert to the fundamental price, F, with some positive adjustment speed b_F. Therefore the excess demand of fundamentalists is modeled by

$$D_t^F = a_F b_F W_t^F (F - P_t). \tag{3}$$

This is in agreement with the idea that fundamentalists tend to hold an asset long when they think it is undervalued and short it when it is perceived to be overvalued. Consistent with Day and Huang (1990) we make the following assumption for the chartists[11]

$$E_t^C[P_{t+1}] = P_t + b_C(P_t - F) \tag{4}$$

with $b_C > 0$ a measure of optimism/pessimism of the chartist. Note that this proposed model is compatible with the following philosophy: when the current price P_t is above the fundamental, an indications of a bull market, chartists expect the price to continue to rise (optimistic chartism). Conversely, when P_t is below the fundamental, an indication of a bear market, chartists expect the price to continue to fall (pessimistic chartism). The excess demand of chartists is thus given by

$$D_t^C = a_C b_C W_t^C (P_t - F). \tag{5}$$

Note that these 'stylized' chartist and fundamentalist demand functions have been chosen to capture the stabilizing force of the fundamental traders and the destabilizing force of the chartists. In regards to the 'trend following' nature of chartists it is possible to add this feature to the

[11]Note that a possible alternative specification for the chartists expectations could be:

$$E_t^C[P_{t+1}] = P_t + b_C\left(P_t - \bar{P}_t\right)$$

where b_C is the expected adjustment speed of the price to the long-run mean level $\bar{P}$. Combining the above two expressions we obtain the demand function

$$D_t^C = b_C W_t^C\left(P_t - \bar{P}_t\right)$$

where we have set $a_C \equiv 1$ without loss of generality. We choose not to use this specification in the following since this would rely on further assumptions about the averaging procedure employed by the chartists.

model through the use of an appropriate switching mechanism, as will be seen later on. In the meantime, to aid our understanding of the above model we can substitute the demands (5) and (3) into the structural equation (1) to obtain

$$P_{t+1} = aF\left(a_F b_F W_t^F - a_C b_C W_t^C\right) + \left[1 + a\left(a_C b_C W_t^C - a_F b_F W_t^F\right)\right] P_t + a e_t \tag{6}$$

from which we can see that the price adjustment parameter a and the reaction parameters a_C and a_F can be incorporated into the other parameters so, without loss of generality, we set $a = a_C = a_F \equiv 1$. We also assume that weights of the chartists and fundamentalists sum to unity, i.e. that $W_t^C = 1 - W_t^F$ for all t. Making these simplifications the above equation becomes

$$P_{t+1} = \left[b_F - (b_F + b_C)W_t^C\right] F + \left[1 - b_F + (b_F + b_C)W_t^C\right] P_t + e_t. \tag{7}$$

As alluded to previously we assume that the relative importance of chartists and fundamentalists is determined by W_t^C and W_t^F and the above representation of the model allows us to see very clearly what effect the time varying weight W_t^C has on the log-price P. We can see from (7) that when $W_t^C = 0$ (fundamentalists dominate the market) the price dynamics reduce to $P_{t+1} = Fb_F + (1 - b_F) P_t + e_t$ which, since $b_F > 0$, is a mean-reverting process.[12] Similarly when $W_t^C = 1$ (chartists dominate the market) the price dynamics reduce to $P_{t+1} = -Fb_C + (1 + b_C) P_t + e_t$ which, since $b_C > 0$, is now an explosive process. Also note the change of sign of the intercept from positive to negative. Obviously somewhere in between, when $W_t^C = \frac{b_F}{b_F + b_C}$, the price dynamics reduce to simply $P_{t+1} = P_t + e_t$, a martingale. Note that the above is precisely the behaviour that heterogenous agent modelers have been using to explain the so-called 'stylized facts' of real financial price dynamics since Frankel and Froot (1986).

As can be seen from the above arguments, the specification of the weights are of crucial importance in this reduced form model, especially when it comes to explaining any real financial time series and its 'stylized facts'. In this paper we use three different specifications for the weights (switching mechanisms) of increasing sophistication and test empirically which mechanism provides the best fit of the model to the empirical data. To aid the interpretation we ground these models in well known and robust empirical estimation procedures.

[12] Furthermore, note that if $b_F > 1$ then we will have 'overshooting' of the the mean and hence the price would exhibit negative auto-correlation.

2.1. **Binary Switching.** To start with we use a simple binary switching mechanism, where W_t^C is either 0 or 1 depending on some criteria. The implications of this is that chartists and fundamentalists are never both active in the market at any given time. The criteria we choose is a simple threshold criteria and to be consistent with the arguments for the chosen demand functions above, we assume that chartists are dominant in the market when the price is above some fixed threshold level, $\bar{P}$, and that fundamentalists dominate when the price is below this level, hence

$$W_t^C = I(P_t - \bar{P} > 0) \tag{8}$$

where $I(\cdot)$ is the indicator function with $I(A) = 1$ when A is true and zero otherwise. A natural choice for the critical level in (8) would be $\bar{P} = F$ but since the fundamental price appears difficult to define in the gold market we choose to endogenise this critical level using a well-known econometric model (TAR).[13]

Note that to our knowledge the above binary switching mechanism has not seen elsewhere in the heterogeneous agents literature. The reason for this is most likely that, with a threshold switching mechanism and the chartist demand function defined as (5), we do not have a restoring force on the price once the price rises above the threshold. Specifically, for b_C and b_F both positive then the price is driven to fundamentals from below by the fundamentalists and then, once it reaches the threshold, the chartists are allowed to increase the price even further, in an unbounded way, as the price continues to rise further and further above fundamentals.[14] This is a deficit of the model which will be improved upon in the smooth switching model outlined next.

2.2. **Smooth Switching.** The second mechanism we propose allows both types of agents to be active in the market at the same time, hence the weight W^C can now take any value in the interval $[0, 1]$. Clearly there are many different specifications one can use for the functional form of this switching function but it has become standard in the HAM literature to use a logistic

[13]This specification is also consistent with the fact that speculation involving selling the asset short is often prohibitively expensive. The implication of the above is that both fundamentalists and chartist only go long (i.e. buy the asset), the fundamentalist below the threshold and the chartist above.

[14]Note that this may not necessarily be a bad model to describe what is happening in the gold market today.

function of some sort.[15] He and Westerhoff (2005) propose the following form for the switching

$$W_t^C = \frac{1}{1+\gamma(F-P_t)^2} \tag{9}$$

where γ is sometimes called the 'intensity of choice' parameter and γ^{-1} the 'status quo bias' (see Kahneman et al., 1982). This bias can be interpreted as the inclination of investors to stick to their current strategy even though objective measures suggest they should switch. The motivation behind the above formulation is that, as the price gets further from fundamentals, even the chartist know that the prices will revert to fundamentals at some point (every bubble will burst) and so the market environment becomes too risky for the chartists, motivating them to switch strategies, or possibly leave the market for less risky alternatives. The reduced chartist activity then allows fundamentalists to drive prices to more moderate values, thus reenforcing the chartists fears. The above mechanism provides a restoring force on the price process, which was lacking in the binary switching mechanism above.

However, the problem with (9) in the context of the gold market is that the fundamental price F is simply not known. In the absence of a fundamental price, we propose the following alternative specification, which remains compatible with the above motivation, but involves an observable proxy for the 'risk' of investing for such chartists

$$W_t^C = \frac{1}{1+\gamma\displaystyle\sum_{j=0}^{N}(P_{t-j}-P_{t-j-1})^2}. \tag{10}$$

Here the term $\sum_{j=0}^{N}(P_{t-j}-P_{t-j-1})^2$, i.e. the sum of the square of the last N period returns, is a proxy for the recent volatility of the gold market. Hence chartists are more active (W_t^C is higher) if the volatility of the gold price P is not too high. If the volatility is too high, chartists view the market conditions as too risky and reduce their activity.

Yet another, perhaps more intuitive, switching specification is based on the past trend in the price of gold. The weight of chartists is given as follows

$$W_t^C = \frac{1}{1+\exp(-\gamma(P_t-P_{t-k}))}. \tag{11}$$

[15]Note this was first proposed by Manski and McFadden (1981) in the context of discrete choice models.

where k determines the length of the period which is used by chartists to assess whether there is a clear positive or negative trend. For example, if $k = 12$, chartists use the past twelve months to identify a trend.

Finally, note that, from an empirical standpoint, the γ entering into these weights makes the estimation problem highly nonlinear. However, if we fix γ and estimate the linear model using maximum likelihood then we can search over all values of γ for the global maximum of the likelihood function. Note that we provide specific graphs that document this grid search in the empirical section below.

The above two model specifications use directly observable variables such as the log-price P. An alternative is to estimate the weights for chartists and fundamentalists as the outcome of an unobservable underlying process via a Markov regime-switching model. This is the procedure we adopt for our final model specification, details of which can be found in the following section. Finally, for all of these approaches, a particularly interesting outcome of the estimation will be to provide estimates of the periods in which chartists and fundamentalists are most active in the gold market.

3. Estimation Framework

This section introduces the econometric methodology to estimate the heterogeneous agents models derived above. We describe the models following Teräsvirta et al. (1994). The heterogeneous agents model given by equation (7) plus either (8), (10) or (11) implies a nonlinear time-series model with two regimes. The standard switching regression model with two regimes is defined as follows

$$y_t = (\boldsymbol{\phi}_1' \boldsymbol{z}_t + \varepsilon_{1t}) I(s_t \leq c) + (\boldsymbol{\phi}_2' \boldsymbol{z}_t + \varepsilon_{2t})(1 - I(s_t \leq c)) \tag{12}$$

where $\boldsymbol{z}_t = (\boldsymbol{w}_t', \boldsymbol{x}_t')'$ is a vector of both endogenous ($\boldsymbol{w}_t$) and exogenous ($\boldsymbol{x}_t$) explanatory variables with $\boldsymbol{w}_t = (1, y_{t-1}, ..., y_{t-p})'$ and $\boldsymbol{x}_t = (x_{1t}, ..., x_{kt})'$, where p and k are both integers. In addition s_t is an observable switch variable, and $I(\cdot)$ is the indicator variable. Finally, $\boldsymbol{\phi}_1'$, $\boldsymbol{\phi}_2'$ and c are parameters to be estimated where c is a threshold parameter.

When $\boldsymbol{x}_t$ is absent and $s_t = y_{t-d}$, $d > 0$, equation (12) becomes the Self-exciting Threshold Autoregressive (SETAR) model. The SETAR model with no exogenous variable $\boldsymbol{x}_t$, one lag

$p = 1$ and $d = 1$ can be written as

$$y_t = (\phi_{11} + \phi_{12} y_{t-1} + \varepsilon_{1t}) I(y_{t-1} \leq c) + (\phi_{21} + \phi_{22} y_{t-1} + \varepsilon_{2t})(1 - I(y_{t-1} \leq c)). \tag{13}$$

The indicator variable $I(\cdot)$ in the SETAR model governs the switch from regime 1 to regime 2 and vice versa. The switching depends on the lagged level of the dependent variable, i.e. y_{t-1}.

An alternative threshold is given by a certain event in time or a date partitioning the sample into two sub-sample periods. In this specification, the indicator variable is defined as $I(t \leq t^*)$ where t^* is a specific date. Since the threshold parameter has to be estimated using a grid, estimation of switching regression models has to be carried out by a set of regressions (see Tong, 1990).

The switching in the above models is abrupt since it is determined by the indicator variable $I(\cdot)$. A smooth switching can be modeled with the Smooth Transition Autoregressive (STAR) model and given by

$$y_t = (\phi_1' y_{t-1} + \varepsilon_{1t}) G(\gamma, c, s_t) + (\phi_2' y_{t-1} + \varepsilon_{2t})(1 - G(\gamma, c, s_t)) \tag{14}$$

where $G(\cdot)$ is the function that determines the transition. The logistic transition function has the general form

$$G(\gamma, c, s_t) = (1 + \exp\{-\gamma(s_t - c)\})^{-1}. \tag{15}$$

Note that $0 \leq G(\gamma, c, s_t) \leq 1$.

An alternative specification for G is

$$G(\gamma, s_t) = (1 + \gamma s_t)^{-1} \tag{16}$$

where s_t is an endogenous or exogenous variable governing the switch from one regime to the other. Note that for this specification we require γ and s_t to have the same sign (i.e. both positive or both negative) to ensure that $0 \leq G(\gamma, s_t) \leq 1$. Recall that we choose not to consider exogenous variables in this paper, only endogenous ones, i.e. lagged log-prices, simple or squared log-price differences, thus providing a 'classical' time-series analysis.

The above models use observable switch variables s_t. An alternative class of models are given by Markov-switching regression models in which the observable switch variable is replaced by an unobservable discrete stochastic variable θ_t that can assume r different values $\{\nu_1, \ldots, \nu_r\}$. The probabilities of a transition from one regime i to another j is denoted

$$p_{ij} = \Pr\{\theta_t = \nu_j | \theta_{t-1} = \nu_i\}, \quad i, j = 1, \ldots, r. \tag{17}$$

The Markov-switching regression model for two regimes ($r = 2$) is thus defined by

$$y_t = (\boldsymbol{\phi}_1' \boldsymbol{z}_t + \varepsilon_{1t}) I(\theta_t = \nu_1) + (\boldsymbol{\phi}_2' \boldsymbol{z}_t + \varepsilon_{2t})(1 - I(\theta_t = \nu_1)). \tag{18}$$

4. Empirical Analysis

The presentation of the econometric framework described in the previous section closely follows Teräsvirta et al. (1994). The model

$$y_t = (\phi_{11} + \phi_{12} y_{t-1} + \varepsilon_{1t}) I(y_{t-1} \leq c) + (\phi_{21} + \phi_{22} y_{t-1} + \varepsilon_{2t})(1 - I(y_{t-1} \leq c)) \tag{19}$$

can be easily transformed into a typical HAM presentation as provided in equation (7) by setting y_t to the log-price P at $t+1$ and the indicator function I equal to W^F ($I = W^F$) and hence $(1 - I) = (1 - W^F) = W^C$.

The empirical analysis of this section contains a descriptive analysis of the data and a presentation and interpretation of the estimation results.

4.1. **Data.**

We use monthly data of the price of gold in US dollars per troy ounce as quoted on the London Bullion Market in the morning (A.M. official). The sample period is January 1980 until December 2010. We use mid-month quotes to calculate the monthly returns (log-price changes). The number of observations is $T = 372$.

Figure 1 presents the price of gold and the return of gold based on the log-price changes. The time-series plot illustrates the strong positive trend from 2005-2010 associated with a higher volatility in that period.

*** Insert Figure 1 about here ***

The average price for the 31-year period is 455 US dollars and the standard deviation is 217 US dollars. The corresponding average monthly return is 0.21 percent and the standard deviation is 5.7 percent. The skewness is slightly negative (-0.043) and the kurtosis is 8.45.

These statistics suggest that gold returns are not normally distributed and exhibit the typical characteristics of financial time-series, especially fat tails. The latter is also evidenced in Figure 2 which shows a histogram of gold returns.

*** Insert Figure 2 about here ***

The next sections present the econometric results.

4.2. **Threshold Autoregressive (TAR) Model.** This section presents the estimation results of the threshold autoregressive (TAR) model. The parameter vectors $\boldsymbol{\phi}_1'$ and $\boldsymbol{\phi}_2'$ are estimated for a set of regressions given by different values of the threshold c. We use a grid search for c from $c = \min(\ln P)$ to $c = \max(\ln P)$ with an initial step size of 0.01 and refined steps of 0.005, 0.0025 and 0.00125. The threshold, the goodness of fit and the parameter estimates 'converged', i.e. did not change when the grid was refined from 0.0025 to 0.00125. Figure 3 shows the changing goodness of fit for different values of c, with a starting value of $\ln P = 6$ and a step size of 0.00125. The figure demonstrates why it is not straightforward to estimate the parameter c due to multiple local maxima and regions in which the objective function to maximize is flat. The full estimation results are presented in Table 1 and the resulting regime-changes are illustrated in Figure 4.[16]

*** Insert Figure 3 about here ***

*** Insert Figure 4 about here ***

The table shows that the optimal threshold is estimated at 694.02 US dollars or a log price of 6.5425 with an R-squared of 0.041. A relative assessment of the model fit with respect to one regime and only one representative agent is discussed in section 4.5. Generally, returns are difficult to explain hence a low R-squared is not surprising. The parameters ϕ_{11} and ϕ_{12} represent the time-series properties in the lower price regime and ϕ_{21} and ϕ_{22} represent the

[16]An alternative model using a temporal threshold rather than a spacial threshold (with the indicator variable $I(t \leq t^*)$ for some threshold time t^*) found t^* to be June 2006 with the period after this time characterized by a non-negative AR coefficient, indicating the presence of a unit-root and hence no mean reversion, suggesting chartists where possibly active in the market during that time. Conversely, the pre-June 2006 period is characterized by relatively strong mean reversion, pointing to the strong presence of fundamentalist behaviour.

Table 1. **Estimation Results: Self-Exciting Threshold Autoregressive Model (SETAR)**

Model: $\Delta P_{t+1} = (\phi_{11} + \phi_{12}P_t + \varepsilon_{1,t+1})I(P_t \leq c) + (\phi_{21} + \phi_{22}P_t + \varepsilon_{2,t+1})(1 - I(P_t \leq c))$

	Coef.	**Std. Err.**	**t-stat**	**prob.>|t|**
ϕ_{11}	0.1955	0.0841	2.32	0.02
ϕ_{12}	-0.0329	0.0141	-2.33	0.02
ϕ_{21}	0.6855	0.3122	2.20	0.03
ϕ_{22}	-0.0968	0.0455	-2.13	0.03
c ($\ln P$, P)	6.5425	694.0195		
R-squared	0.04052			

higher price regime (prices above the estimated threshold). The mean of the lower and higher price regimes are US dollar 380.79 (log-price calculated as $(-0.1955)/(-0.0329) = 5.94$) and US dollar 1189.88 ($(-0.6855)/(-0.0968) = 7.08$) respectively. These coefficient estimates imply that there is both mean reversion in the lower price regime ($\phi_{12} = -0.0329$) and in the higher price regime ($\phi_{22} = -0.0968$) with an apparently stronger mean reversion in the higher price regime. The larger coefficient estimate ϕ_{22} compared to ϕ_{12} indicates that the mean-reversion is more profound but the t-statistic suggests that this difference is not statistically significant. The relative strong mean reversion and its implication that fundamentalists are more active in the high-price regime is counterintuitive since the period from 2002-2010 is characterized by a strong positive trend and a significant portion of the high-price regime falls into this period. An explanation for this finding is the observation that all three regimes when the price is above the threshold of US dollar 694 can be characterized by a price correction. This effect is most pronounced in the aftermath of the global financial crisis where the price of gold fell from around US dollar 1000 in March 2008 to around US dollar 730 in November 2008. The large drop in the value of gold is remarkable as it fully overlaps with the peak of the crisis in October 2008.

The regime-specific average values imply that prices mean revert from below and from above. In other words, if the price is below the average (e.g. 5.94 or US dollar 380 in the lower price regime), prices revert to the mean from below consistent with a situation in which investors buy the asset. In contrast, if the price is above the average, prices revert to the mean from above, i.e. investors sell the asset.

4.3. **Smooth-transition Autoregressive (STAR) Model.** This section presents the smooth-transition autoregressive models for two alternative switching specifications. The first model assumes that regime changes depend on the lagged volatility of the log-price differences as given by

$$G_1(\gamma, P) = (1 + \gamma(P_t - P_{t-1})^2)^{-1} \tag{20}$$

This smooth-transition specification assumes that low and high volatility regimes determine whether chartists or fundamentalists are more active in the market.

A second, alternative, specification assumes that the transition is determined by past log-price differences as follows

$$G_2(\gamma, P) = (1 + \exp(-\gamma(P_t - P_{t-k})))^{-1} \tag{21}$$

for some lag k. This model leads to a transition from one regime to the other if the difference of price changes is large. The specification is not only different economically but also technically as the log-price difference between $t-k$ and t can be both positive and negative. The larger the difference is (towards infinity), the larger is G_2 (closer to one) for $\gamma > 0$. On the other hand, if the difference is very close to zero then $G_2 \approx 0.5$ and as the difference becomes increasingly negative then the smaller is G_2 (closer to zero). This specification can be mapped to the heterogeneous agents model in which chartists or trend-chasers are more active if there is a positive trend in a market ($G_2 \to 1$) and fundamental traders are more active if there is no trend or a negative price trend. The increasing importance of fundamental traders in bear markets (negative trend) can be explained with the larger costs of speculating on a negative trend or holding short positions relative to speculating on a positive trend or a bull market. This implies that chartists are more likely to be present in bull markets than in bear markets.

The results of the first and second model are presented in Tables 2 and 3, respectively.

Table 2 illustrates that the best model fit is obtained for $\gamma = 139$ (see also Figure 6) and two distinct regimes with a weakly explosive process in the low volatility regime given by ϕ_{11} and ϕ_{12} and a mean-reverting process in the high volatility regime. The coefficient estimates represent a situation in which chartists or trend-chasers drive the price away from the fundamental value or the long-run mean in a low-volatility regime, i.e. chartists are more active in tranquil conditions. In contrast, the coefficient estimates in the high volatility regime are consistent with a situation

TABLE 2. **Estimation Results (1): Smooth-transition Autoregressive Model (STAR)**

Model: $\Delta P_{t+1} = (\phi_{11} + \phi_{12}P_t + \varepsilon_{1,t+1})G_1(\gamma, P) + (\phi_{21} + \phi_{22}P_t + \varepsilon_{2,t+1})(1 - G_1(\gamma, P))$
$G_1(\gamma, P) = (1 + \gamma(P_t - P_{t-1})^2)^{-1}$

	Coef.	**Std. Err.**	**t-stat**	**prob.>|t|**
ϕ_{11}	-0.1075	0.0698	-1.54	0.12
ϕ_{12}	0.0185	0.0116	1.60	0.11
ϕ_{21}	0.3780	0.1980	1.91	0.06
ϕ_{22}	-0.0626	0.0319	-1.96	0.05
γ	139			
R-squared	0.0157			

in which fundamentalists enforce mean-reversion to the regime-specific mean. Since this value (6.04 or US dollar 419) is relatively low, then prices above this value will revert from above, a situation well described by fundamentalists selling the asset and thereby establishing mean reversion.

The regime-switching through time is given by $G_1(\cdot, \cdot)$ and illustrated in Figure 5. Note that it appears that fast switching between the two regimes is required to fit the data to the model, in agreement with the findings of Amilon (2008).

Figure 6 illustrates that the estimate of γ is obtained through a grid search in which the likelihood function (or R-squared) is calculated for each possible value of γ. The value that maximizes the likelihood function is chosen as the best available estimate. The figure indicates that there is a region in the neighborhood of the optimal γ (= 137) which is rather flat. This outcome, which is a typical phenomenon encountered with the estimation of these models,[17] is the justification for the alternative grid search or two-stage estimation strategy.

*** Insert Figure 6 about here ***

*** Insert Figure 5 about here ***

[17]See Teräsvirta (1994).

Table 3. **Estimation Results (2): Smooth-transition Autoregressive Model (STAR)**

$$\text{Model: } \Delta P_{t+1} = (\phi_{11} + \phi_{12}P_t + \varepsilon_{1,t+1})G_2(\gamma, P) + (\phi_{21} + \phi_{22}P_t + \varepsilon_{2,t+1})(1 - G_2(\gamma, P))$$
$$G_2(\gamma, P) = (1 + \exp(-\gamma(P_t - P_{t-12}))^{-1}$$

	Coef.	**Std. Err.**	**t-stat**	**prob.>|t|**
ϕ_{11}	-0.0288	0.0629	-0.46	0.65
ϕ_{12}	0.0061	0.0102	0.60	0.55
ϕ_{21}	0.2038	0.1163	1.75	0.08
ϕ_{22}	-0.0354	0.0197	-1.79	0.07
γ	37.95			
R-squared	0.023			

Table 3 presents the estimation results for the smooth-transition given by the lagged price change or past trend ($P_t - P_{t-12}$). We assume one calendar year (twelve months) as the 'natural' time frame which is used by chartists to identify a trend in the market. As the 12-month period is arbitrary to some degree we estimated the model with shorter and longer time periods with similar results for small variations and a substantial loss in the fit of the model for significantly shorter or longer time periods.

The larger (smaller) the lagged price change or trend, the larger (smaller) is G_2 for $\gamma > 0$. This model structure implies that the parameters ϕ_{1i} for $i = 1, 2$ govern the evolution of the time-series if the lagged price changes are large (i.e. positive) and ϕ_{2i} for $i = 1, 2$ determine the evolution for lagged price changes that are small (i.e. negative). The results indicate that the regime of large past positive price changes is represented by insignificant coefficient estimates. The estimates imply that the null hypothesis of a unit root in the time-series can not be rejected. On the other hand, the coefficient estimates for the regime of small price changes can be characterized by a mean-reverting process. The latter is consistent with more active fundamental traders who establish mean reversion or a correction of deviations from the regime-specific mean if there is no clear positive trend in the asset, i.e. no bull market. The results for large price changes $G_2 \to 1$ (e.g. a bull market) can be interpreted as a situation in which chartists or trend-chasers are more active as the coefficient estimates do not indicate mean-reversion in such a regime.

Figure 7 illustrates the time-varying weights $G_2(\cdot,\cdot)$. Note that here we see a much slower rate of switching compared to $G_1(\cdot,\cdot)$ providing strong evidence of dominant chartists or fundamentalist at any given time. For example, it can clearly be seen that chartists appeared to have dominated the market during the period 2002-2006 since G_2 was close to one for the majority of this period.

*** Insert Figure 7 about here ***

4.4. **Markov Regime-switching Model.** This section presents the results of a model based on an unobservable process in contrast to the models estimated above. All variables including the model's variance depend on a state $I(\theta)$ which is unobserved. Here ν_1 denotes the low variance regime or state and ν_2 the high variance regime.

TABLE 4. **Estimation Results: Markov Regime-switching**

Model: $p_{ij} = \Pr\{\theta_{t+1} = \nu_j | \theta_t = \nu_i\}, \quad i,j = 1,2$

$$\Delta P_{t+1} = (\phi_{11} + \phi_{12}P_t + \varepsilon_{1,t+1})I(\theta_{t+1} = \nu_1) + (\phi_{21} + \phi_{22}P_t + \varepsilon_{2,t+1})(1 - I(\theta_{t+1} = \nu_1))$$

		Coef.	Std. Err.	t-stat.	Prob.
state 1 (low variance regime)	ϕ_{11}	-0.0857	0.0378	-2.2672	0.0200
	ϕ_{12}	0.0145	0.0063	2.3016	0.0200
state 2 (high variance regime)	ϕ_{21}	0.0966	0.1725	0.5600	0.5800
	ϕ_{22}	-0.0144	0.0278	-0.5180	0.6100
Transition probabilities	0.97	0.10			
	0.03	0.90			
Expected duration of regime (state 1)		282.81	months		
Expected duration of regime (state 2)		89.19	months		

Table 4 presents the estimation results of the Markov regime-switching model. The estimates show that the low variance regime can be characterized by an explosive process ($\phi_{12} > 0$) and the high variance regime can be characterized by a mean-reverting process ($\phi_{22} < 0$). This finding is consistent with chartists being more active in the lower variance regime establishing price trends that lead away from the long-run mean and fundamental traders being more active in the higher variance regime and causing prices to mean-revert.

We note that the result obtained with the Markov regime-switching model based on an unobservable process is similar to the findings achieved with an observable process as used by the STAR model reported in Table 2.

The probability of being in a specific regime in the Markov-switching model can be easily plotted but almost perfectly resembles the probability of being in a positive trend regime (switching function G_2) in the second STAR model (see Figure 7). The fact that the probability of being in a chartist-regime has been close to one over the last 10 years is strong evidence for the fact that chartists have been propping up the price as opposed to fundamentalists in recent times. Of particular interest is the period from mid-2009 until now which has seen an extreme rise in price and which is accompanied by a unit probability of being in a chartist dominated regime.

4.5. **Specification Issues.** The goodness of fit of the models estimated above is relatively low but expected given the models are based on log price differences. The model fit measured with R-squares would be close to 100% if levels were used. The models above are estimated with ordinary least squares but can also be estimated with Maximum Likelihood and yield the same results. The latter point is representative of the general objective of this paper: use relatively simple and parsimonious models to estimate a heterogeneous agents model. We also chose to estimate a set of regressions and use a grid search to obtain the switching variables as an alternative to Maximum Likelihood. The objective function of the TAR model well-illustrated that coefficient estimates can be sensitive to the starting values if all coefficient estimates are estimated simultaneously. In addition, given the structure of the heterogenous agents model we assumed two regimes in all econometric models estimated. The coefficient estimates of a model with only one regime yield an R-squared of 0% and statistically insignificant coefficient estimates of 0.00047 for the autoregressive term and -0.00076 for the intercept. This result further justifies the specification of two regimes.

5. Summary and Concluding Remarks

This study analyzed the price of gold by following a pure time-series econometrics approach. The aim being to empirically test a heterogenous agents model with fundamentalists and chartists in the gold market. A theoretical model is used which posits that the dominance of fundamentalists in the market leads to mean-reverting prices and that the dominance of chartists leads to explosive prices. Furthermore, speculators switching between fundamentalist and chartist strategies can account for the different observed price dynamic in the gold market over the last 30 years. More specifically, we estimate three different econometric models with different switching mechanisms: (i) a binary switching model, (ii) a smooth-transition switching model and (iii) a Markov-switching model. The models can be classified into observable and unobservable switching variables. We estimate a threshold and assume that the switching depends on the price of gold. Alternatively, we test whether the switching between fundamentalists and chartists depends on the volatility of the price of gold, a common assumption in the theoretical literature. Finally, we test whether the switching is determined by the most recent trend in the price of gold or the potential profit chartists could have made if they had invested in the asset twelve months ago. The empirical results show that both the volatility and the past trend are significant switching variables. The results are confirmed by the estimation results of the Markov-switching model which uses unobservable switching variables. The graphical analysis of the changing weight or role of fundamentalists and chartists further indicates that chartists appear to be responsible for the substantial changes in the price of gold in recent years.

Recent events, especially the global financial and economic crisis in 2008 and the current sovereign debt crisis offer an alternative explanation for the high price of gold. However, we believe that an almost continuously rising price of gold from around 400 US dollar in 2004 to more than $1,500$ in 2011 cannot be explained with the behaviour of fundamentalists only. This paper is a starting point to better understand the time-series characteristics of the price of gold and to associate these characteristics with the behaviour of fundamentalists and chartists within a theoretical model which can be tested empirically.

There are several issues which should be addressed in future studies. First, the role of the fundamental value is not fully understood especially in the context of commodities where a fundamental price is rather opaque compared to the stock market or the housing market. Second, this study focussed on prices but did not model the volatility and the influence of chartists and fundamentalists on the volatility of the price of gold. Third, the role of exogenous variables like the US dollar or the volatility of the stock market could be analyzed. The theoretical model used in this paper may be simple but it is tested empirically and thus provides insights that go beyond a pure simulation study.

FIGURE 1. The gold price (solid line) and corresponding monthly returns (dotted line).

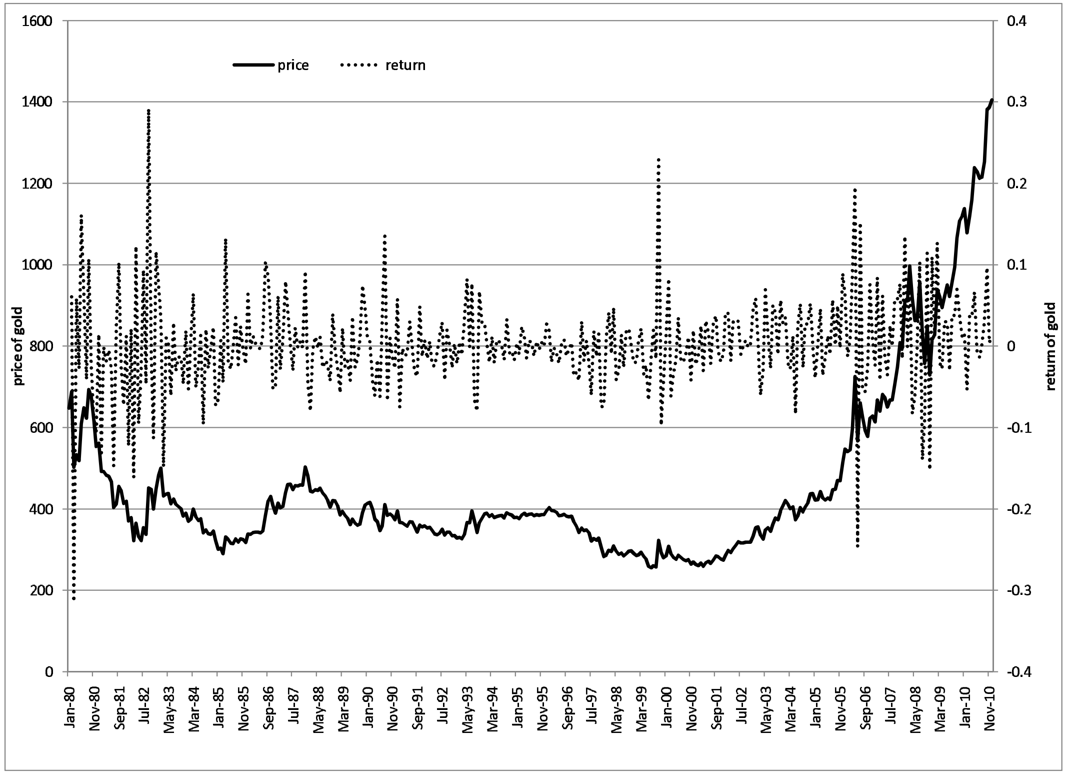


FIGURE 2. Histogram of gold returns (log-price changes).

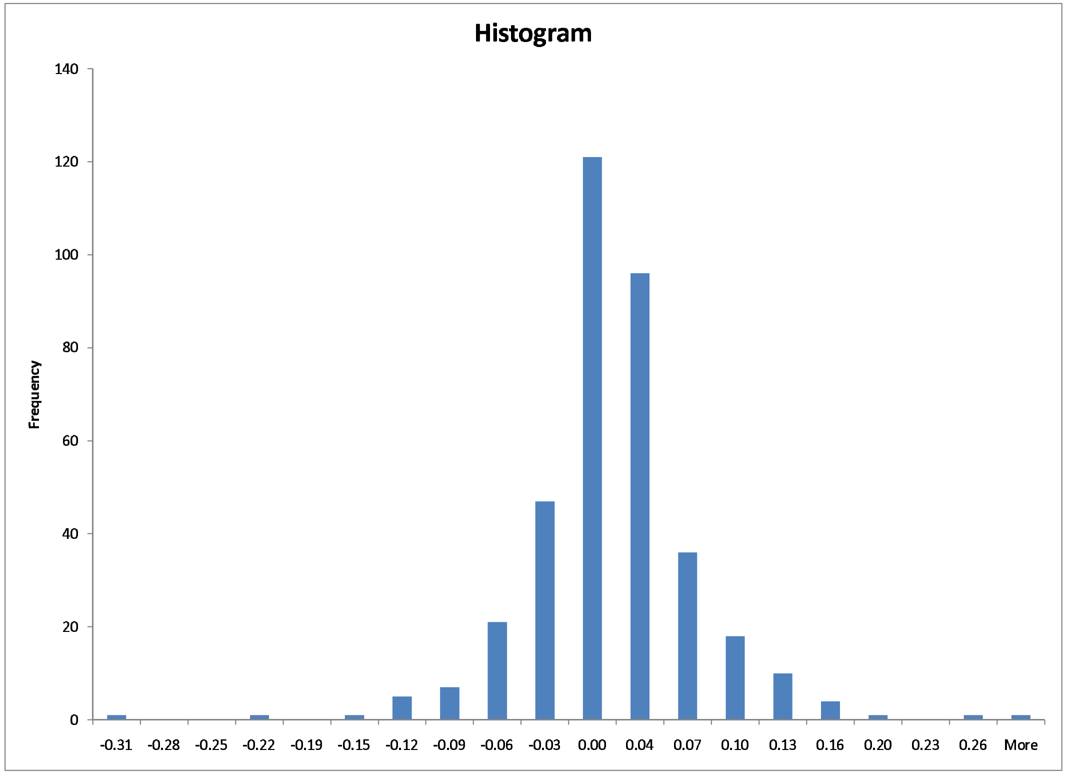


Figure 3. TAR model fit (R-squared) for different thresholds, c.

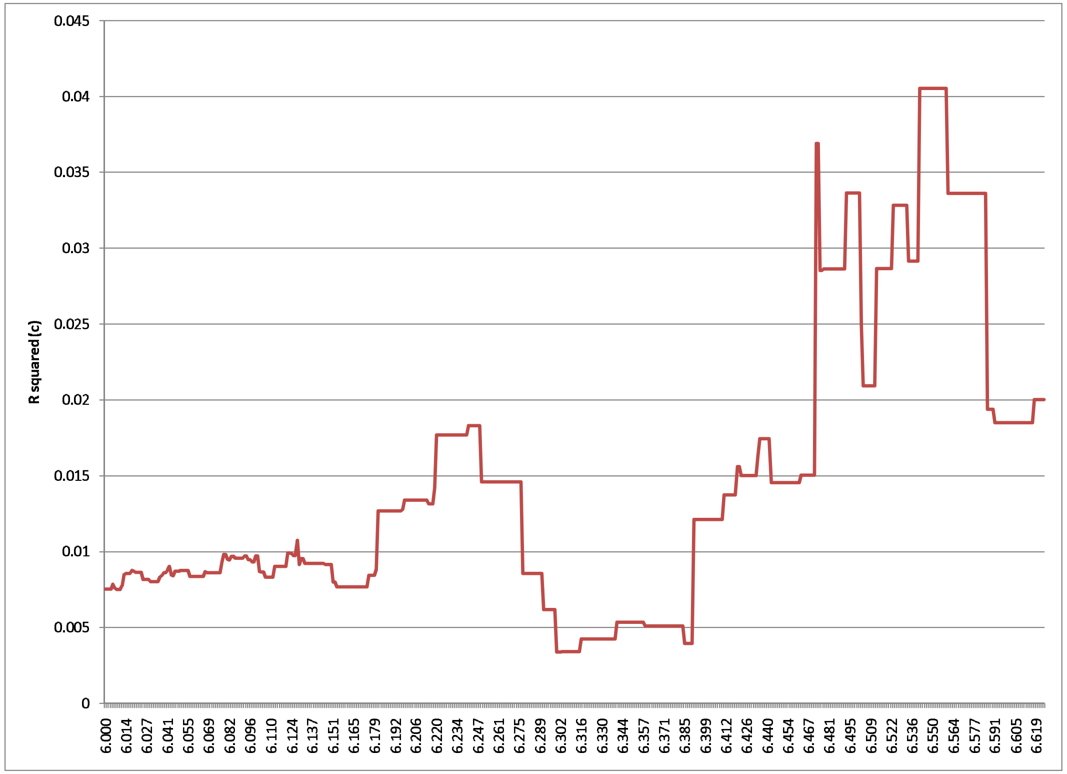


Figure 4. TAR Model regime estimates.

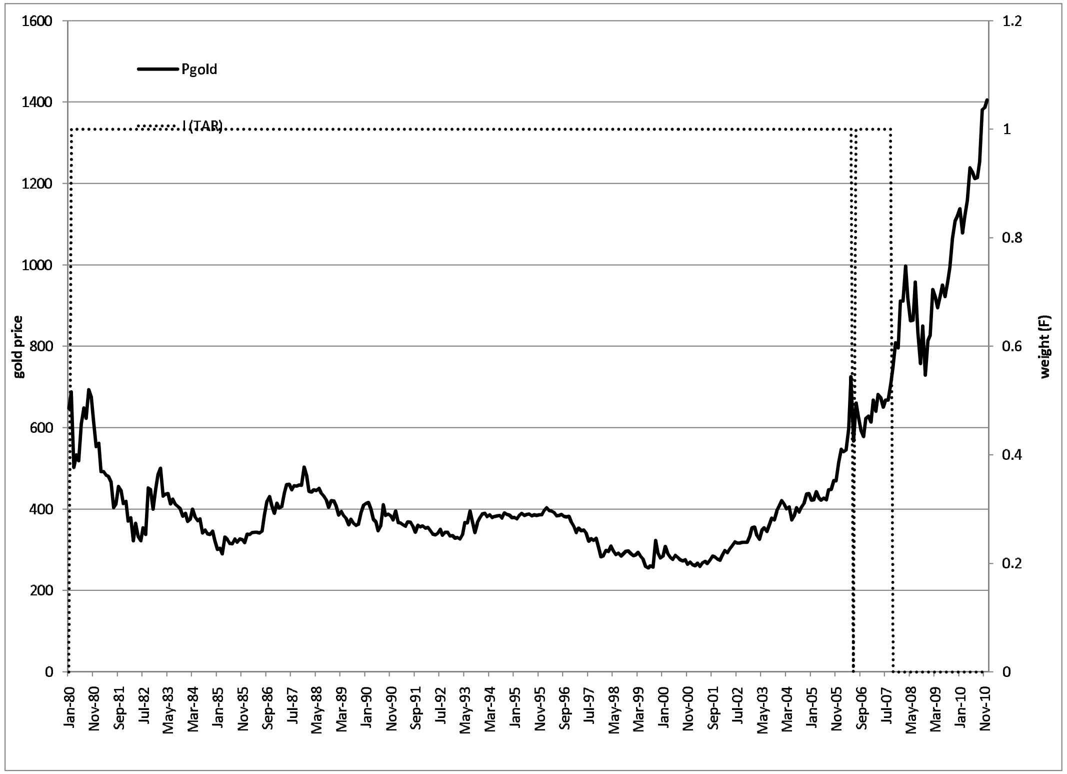


Figure 5. Time-varying weights or switching variable $G_1(\gamma, P)$ (dotted line) and the log price of gold (solid line).

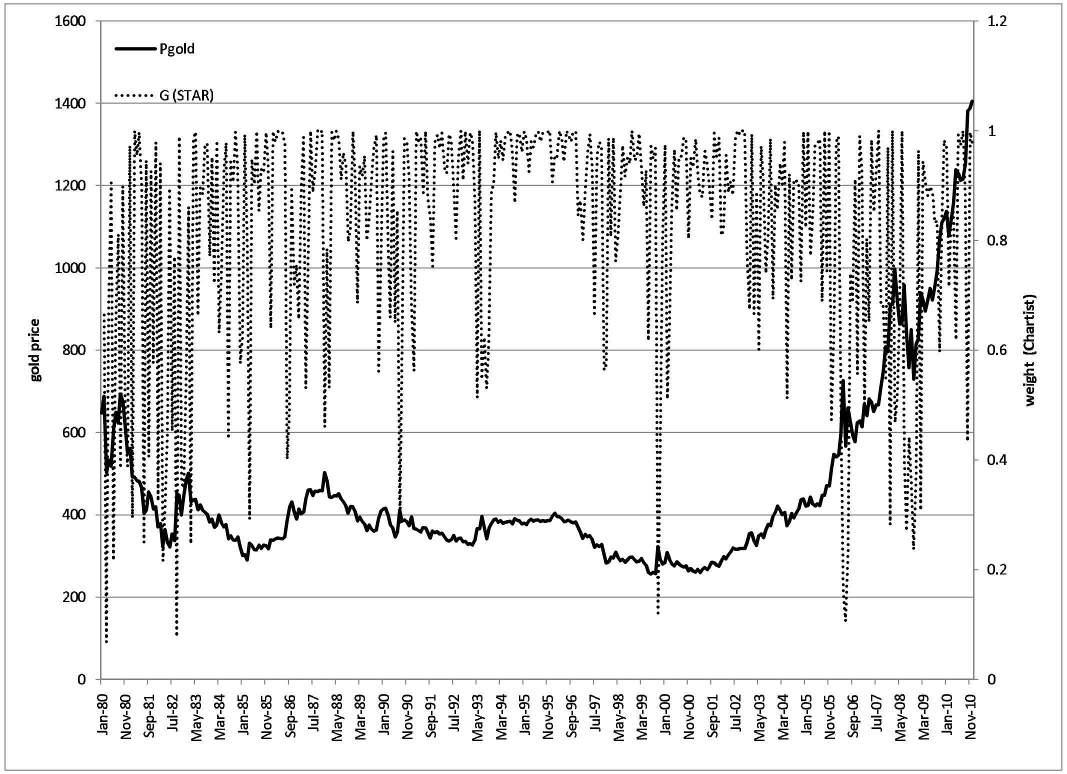


Figure 6. Grid search for γ using $G_1(\gamma, P)$.

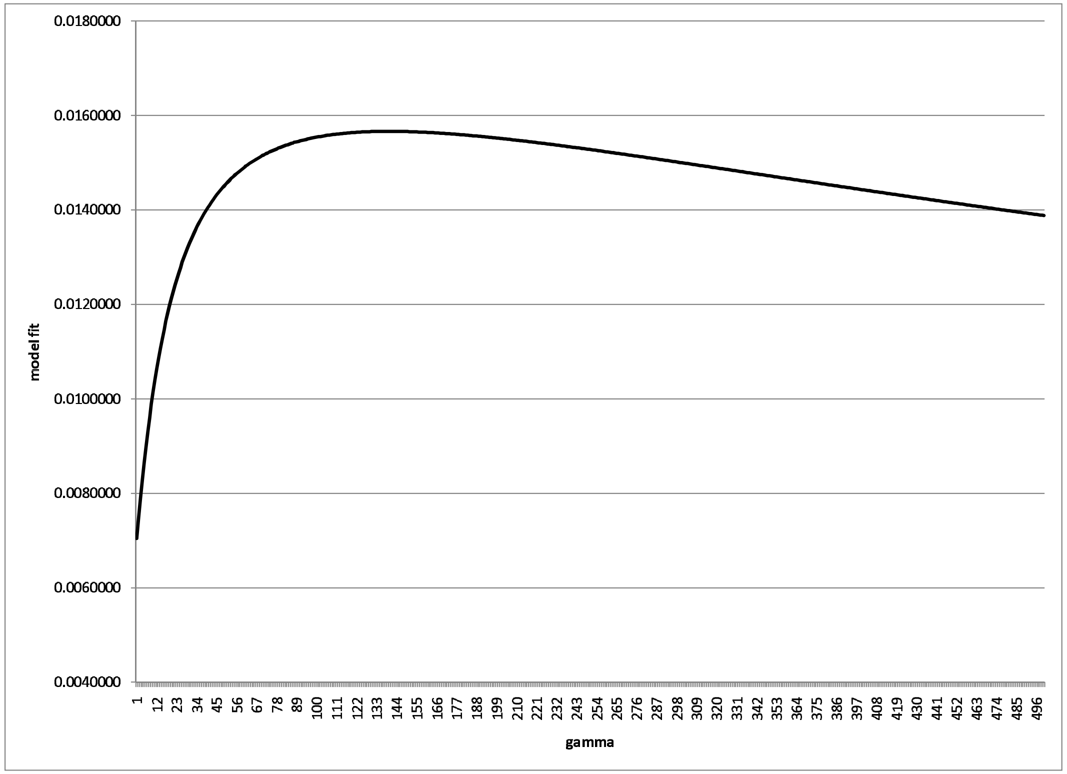
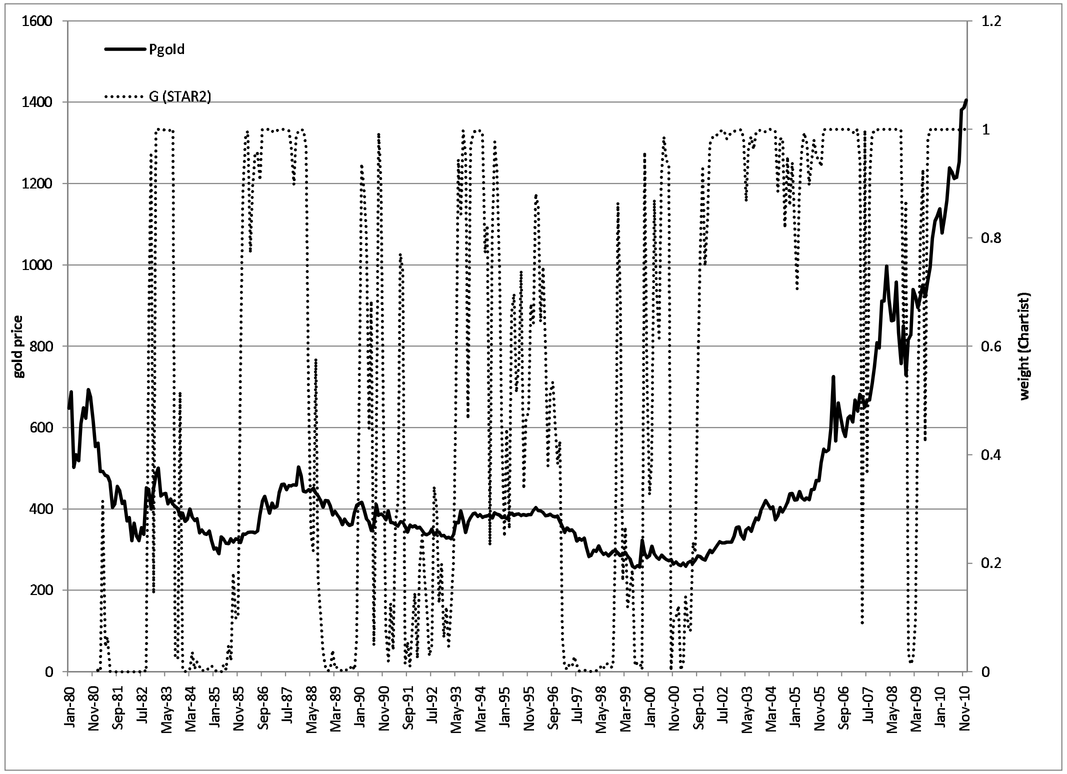


FIGURE 7. Time-varying weights or switching variable $G_2(\gamma, P)$ (dotted line) and the log price of gold (solid line).



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